REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* 23-03-2007	**2. REPORT TYPE** Journal Article	**3. DATES COVERED** *(From - To)*

4. TITLE AND SUBTITLE

Effect of Thermal Conductivity on the Knudsen Layer at Ablative Surfaces (Postprint)

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

L. Pekker (ERC, Inc.); M. Keidar (Univ. Mich.), J.-L. Cambier (AFRL/PRSA)

5d. PROJECT NUMBER

5e. TASK NUMBER
50260542

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZSA
10 E. Saturn Blvd.
Edwards AFB CA 93524-7680

8. PERFORMING ORGANIZATION REPORT NUMBER

AFRL-PR-ED-JA-2007-189

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RZS
5 Pollux Drive
Edwards AFB CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S NUMBER(S)
AFRL-PR-ED-JA-2007-189

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited (PA #07151A).

13. SUPPLEMENTARY NOTES

Published in JOURNAL OF APPLIED PHYSICS 103, 034906 2008. ©2008 American Institute of Physics

14. ABSTRACT

In this paper we develop an analytical model of Knudsen layer at the ablative wall taking into account the temperature gradient in the bulk gas. The region of validity of the existing models and effect of the temperature gradient on the Knudsen layer properties are calculated.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Dr. Jean-Luc Cambier
a. REPORT **Unclassified**	**b. ABSTRACT** **Unclassified**	**c. THIS PAGE** **Unclassified**	SAR	7	**19b. TELEPHONE NUMBER** *(include area code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

Effect of thermal conductivity on the Knudsen layer at ablative surfaces

L. Pekker,[1,a)] M. Keidar,[2] and J.-L. Cambier[3]
[1]ERC Inc., 10 East Saturn Boulevard., Edwards AFB, California 93524, USA
[2]Department of Mechanical and Aerospace Engineering, The George Washington University, Washington, DC 20052, USA
[3]Air Force Research Laboratory, 10 East Saturn Boulevard, Edwards AFB, California 93524, USA

(Received 9 October 2007; accepted 29 November 2007; published online 13 February 2008)

In this article we develop an analytical model of the Knudsen layer at the ablative wall taking into account the temperature gradient in the bulk gas. The analysis is based on the premise that the temperature gradient in the bulk gas can be taken into account in the velocity distribution function at the outer boundary of the Knudsen layer. The model uses a bimodal velocity distribution function in the Knudsen layer, which preserves the laws of conservation of mass, momentum, and energy and converges to the Chapman–Enskog velocity distribution function at the outer boundary of the layer. The model is applied to polyethylene ablation, for which two cases are considered: (a) the ablation process is due to pure heat conduction to the surface, with no external heating of the ablated surface, and (b) the ablation is due to both the thermal conduction and an external heating of the surface, e.g., vaporization of a metal exposed to laser radiation. The region of validity of the existing models and effect of the temperature gradient on the Knudsen layer properties are calculated. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2838210]

I. INTRODUCTION

The physics of the Knudsen layer, the nonequilibrium (kinetic) layer, formed near the vaporizing (ablative) surface is of great interest for a number of applications, such as capillary discharges,[1,2] plasma thrusters,[3,4] high-pressure discharges,[5] vacuum arcs,[6] electroguns,[7] and laser ablation.[8]

Anisimov[9] was the first to consider details of the vaporization process for a case of vaporization of a metal exposed to laser radiation. He used a bimodal velocity distribution function in the kinetic layer, assuming no absorption of laser radiation in the ablated gas. The primary result of his work was the calculation of the maximal flux of returned atoms to the evaporating surface, which was found to be about 18% of the flux of vaporized atoms. This result was obtained under the assumption that the atom flow velocity is equal to the sound velocity at the external boundary of the Knudsen layer and the temperature of the gas in the equilibrium region (beyond the Knudsen layer) is constant, i.e., no conductive heat flux to the ablative wall surface.

However, in many physical situations, the vapor leaving the nonequilibrium layer cannot be described by using a speed of sound approximation. For example, in ablative capillary discharges, the gas motion in the capillary chamber is not "free"; it is restricted by the capillary wall, leading to a more dense gas (plasma) in the discharge volume and therefore, larger back flux to evaporating surface and smaller flow velocity at the outer boundary of the Knudsen layer. Ytrehus[10] has used the Anisimov and Ansatz bimodal velocity distribution functions in the Knudsen layer to study the effect of bulk gas pressure on downstream vapor flow (half-space evaporation problem). He has calculated the density and temperature jumps over the Knudsen layer, the evaporation mass flux and other parameters of the Knudsen layer as functions of the ratio of the equilibrium vapor pressure to the gas pressure at the outer boundary of the Knudsen layer (gas bulk pressure). In addition, Ytrehus has demonstrated that his analytical results are in substantial agreement with the experimental finding, direct simulation Monte Carlo (DMCS) and numerical solutions of the Boltzmann equations. He has also shown that the differences between the analytical solutions using the Anisimov approximation and the Ansatz (more sophisticated) velocity distribution function are very small.

Beilis[11,12] was the first to consider ablation into a dense plasma. He studied the case of metal vaporization into discharge plasmas in a vacuum arc cathode spot. He concluded that the parameters at the outer boundary of the Knudsen layer are close to their equilibrium values and that the velocity at the outer boundary of the kinetic layer is much smaller than the sound velocity.

Later these models were applied for the case of dielectric ablation into the discharge plasma in the capillary discharge conditions[13,14] and for the case of strong plasma acceleration.[15] All those analytical models neglected the conductive heat flux to the ablative surface. This can be significant because the temperature in the plasma core is assumed in the models to be much greater than the temperature of the ablative surface. In particular, neglecting the conductive heat flux results in the calculated gas temperature at the outer boundary of the Knudsen layer to appear to be smaller than the temperature of the evaporating surface. This consequently leads to the heat flux through the Knudsen Layer being directed upward to the plasma chamber. Therefore, neglecting the conductive heat transfer in the Knudsen layer leads to an inconsistency in all the models[1–7] where the gas (plasma) temperature is larger than the surface temperature of the ablative wall.

[a)]Electronic mail: leonid.pekker.ctr@edwards.af.mil.

However, it is worth noting, that in the case where the external heat flux to the ablating surface is larger than the conduction heat flux, the temperature of the ablating surface might be larger than the gas temperature at the outer boundary of the Knudsen layer. This is the case for the example in Ref. 9 where an externally applied laser radiation source heats the ablating surface, but for which the gas (plasma) is transparent.

Recently many numerical models of evaporation processes were developed based on Monte Carlo simulation[16–18] and numerical solutions of the Boltzmann equations,[19] describing the kinetic layer without any a prior approximation of gas velocity function distribution in that layer. However, as Rose[20] has demonstrated the models using his and Anisimov's bimodal velocity distribution functions in the kinetic layer give results virtually coincident with numerical solutions of the Boltzmann–Krook–Welander equation for evaporation of a monatomic substance with condensation coefficient equal to unity. As shown in Refs. 10 and 16–18 analytical models also are in good agreement with Monte Carlo simulation.

Ideally Monte Carlo simulations should be able to self-consistently describe the conductive heat flux to the ablating surface. However, this will require extending the analysis beyond the Knudsen layer region, making it computationally intensive. Thus, improving the analytical models by including heat conduction into consideration is an important step in developing practical (computationally efficient) solutions for modeling of evaporation processes and plasma discharges coupled to ablative processes, and in improving our physical understanding of the Knudsen layer.

We would also like to point out the recent article by Bond and Struchtrup,[21] in which the authors have included the conduction heat flux in their analytical model of water evaporation. This is a generalized Hertz–Knudsen model of the Knudsen layer, which uses the condensation and accommodation coefficients (probabilities) at the evaporating surface to take into account back-flux effect in the Knudsen layer. However, these coefficients can have a complex dependence on the evaporation conditions such as vapor pressure, temperature, surface conditions, incidence angle, etc., and are usually determined experimentally or by fitting the model with experimental data. In the case of dielectric ablation into plasma, where the ablation parameters vary among a wide range of temperatures and pressures, these coefficients are not experimentally measured. It is worth noting that the classical Hertz–Knudsen model and all its generations assumes no collisions in the Knudsen layer (i.e., does not satisfy conservation of momentum through the Knudsen layer) preventing any "relaxation" in the kinetic (Knudsen) layer, where the velocity distribution function at the ablative surface has to relax (converge) to the bulk gas distribution function at the outer boundary of the Knudsen layer. The analytical models using bimodal velocity distribution functions, such as the one described here, take into account the collisions in the Knudsen layer, satisfy the conservation of momentum through the Knudsen layer, self-consistently calculate the back flux and implicitly assume a condensation coefficient of unity. As has been mentioned earlier, they describe the ablation process with reasonable approximation, and the effect of variable condensation coefficient is not considered there.

In this article we develop an analytical model of the Knudsen layer by considering an appropriate boundary condition in the kinetic formulation that takes into account the gas temperature gradient at a flat gas–wall interface. The region of validity of the existing models and the effect of the temperature gradient on the Knudsen layer properties are calculated. The main impetus of this article is to study the effect of the thermal conductivity on the Knudsen layer formed near the ablated surface. This analysis is based on the premise that thermal conductivity (the temperature gradient) in the gas bulk can be taken into account in the velocity distribution function at the outer boundary of the Knudsen layer. In this article we use such a function obtained by Chapman–Enskog expansion method for solving the Boltzmann equation[22] based on the assumption that the molecular mean-free path is much smaller than characteristic scale of the temperature change. Thus, our model is limited to relatively small values of the temperature gradients.

II. MODEL FORMULATION

Following Anisimov's method,[9] let us write the velocity distribution function in the kinetic layer with the evaporating surface in the following form, Fig. 1

$$f(x,\mathbf{V}) = \delta(x)\cdot f_b(\mathbf{V}) + [1-\delta(x)]\cdot f_u(\mathbf{V}), \tag{1}$$

where

$$f_b(\mathbf{V}) = \begin{cases} \left(\dfrac{1}{\sqrt{\pi}}\right)^3 \exp(-V^2), & V_x > 0 \\ \beta n_1 \left(\dfrac{1}{\sqrt{\pi}V_1}\right)^3 \exp\left(-\dfrac{(V_x-u)^2+V_y^2+V_z^2}{V_1^2}\right), & V_x < 0 \end{cases}, \tag{2}$$

$$f_u(\mathbf{V}) = n_1\cdot f_M(\mathbf{V})\cdot\left\{1-\frac{V_T V_1}{\nu} \times\left[\frac{(V_x-u)}{V_1}\left(\frac{(V_x-u)^2+V_y^2+V_z^2}{V_1^2}-\frac{5}{2}\right)\frac{d}{dx}(\ln T)\right]\right\}, \tag{3}$$

$$f_M(\mathbf{V}) = \left(\frac{1}{\pi V_1^2}\right)^{3/2}\exp\left(-\frac{[(V_x-u)^2+V_y^2+V_y^2]}{V_1^2}\right). \tag{4}$$

Here f_b is the velocity function distribution at the inner boundary of Knudsen layer (at the ablative surface) with Maxwellian vaporization function for $V_x > 0$ and a shifted "back flux" Maxwellian function of the particle for $V_x < 0$ describing the particles incoming to the surface from the gas, where the x axis is normal directed to the wall from the gas chamber; f_u is the Chapman–Enskog velocity distribution function at the outer boundary of the Knudsen layer that takes into account the temperature gradient and directed velocity[22] above the Knudsen layer, as shown in Fig. 1;

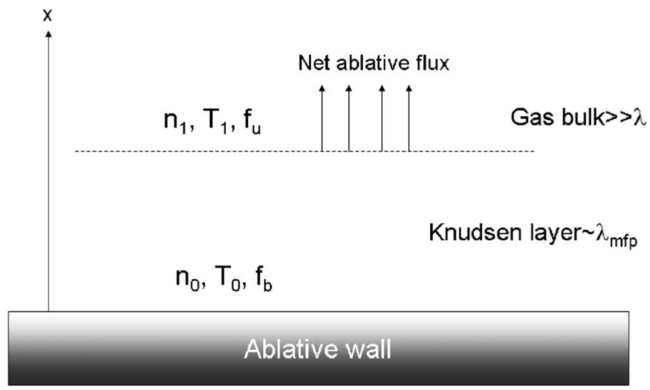

FIG. 1. Schematic representation of the layer structure near the ablative surface.

$kT_0 = mV_T^2/2$ is the temperature of the ablated surface; ν is the collision frequency depending on the temperature and density of the gas; $\delta(x)$ is an unknown function that satisfied the conditions $\delta(0)=1$ and $\delta(\infty)=0$. The parameter β is an unknown variable that must be obtained in the solution; it represents essentially nonequilibrium effects caused by collisions in the Knudsen layer. The number densities are normalized on the equilibrium vapor number density n_0 corresponding to the surface temperature T_0 and all velocities are normalized on V_T. The expression for f_b takes into account the fact that the vaporized atoms have a Maxwellian distribution at a temperature equal to the surface temperature[23] and also assumes that the number density of the evaporated atoms is equal to half of the equilibrium vapor number density at this surface temperature, i.e., reasonable approximations used in all previous bimodal velocity distribution functions models.[9–18] It should be noted that Anisimov's model[9] (as well as all other existing models) employed shifted Maxwellian function at the outer boundary of Knudsen layer ignoring, as mentioned previously, the conduction heat transfer to the ablative surface, i.e., temperature gradient at the edge of the Knudsen layer.

Assuming the conservation laws of mass, momentum, and energy hold at all times within the discontinuity region, through the Knudsen layer, as it has been assumed in all previous models[9–15] (quasisteady state approximation within the Knudsen layer), the following integrals are defined:

$$C_1 = \int_{-\infty}^{+\infty} dV_z \int_{-\infty}^{\infty} dV_y \int_{-\infty}^{+\infty} fV_x dV_x = n_1 u, \tag{5}$$

$$C_2 = \int_{-\infty}^{+\infty} dV_z \int_{-\infty}^{\infty} dV_y \int_{-\infty}^{+\infty} fV_x^2 dV_x = n_1\left(u^2 + \frac{V_1^2}{2}\right), \tag{6}$$

$$C_3 = \int_{-\infty}^{+\infty} dV_z \int_{-\infty}^{\infty} dV_y \int_{-\infty}^{+\infty} fV^2 V_x dV_x = n_1\left[u\left(u^2 + \frac{5V_1^2}{2}\right) - \frac{(V_T V_1)}{\nu}\frac{d(\ln T)}{dx}\frac{5\cdot V_1^3}{4}\right], \tag{7}$$

where values of C_1, C_2, and C_3 obtained at the outer boundary of the Knudsen layer, where δ is equal to zero, and the mass, momentum, and energy fluxes are

$$M_x = mn_0 V_T C_1, \tag{5'}$$

$$P_x = mn_0 V_T^2 C_2, \tag{6'}$$

$$E_x = mn_0 \frac{V_T^3}{2} C_3, \tag{7'}$$

where E_x consists of the two parts: the conduction heat flux and the enthalpy flux of the gas moving with a directed velocity, Eq. (7). Taking into account that integrals C_1, C_2, and C_3 are preserved through the Knudsen layer and they should be independent on $\delta(x)$, we obtain the following equations corresponding to C_1, C_2, and C_3:

$$\frac{1}{2\sqrt{\pi}} = C_1 - n_1\beta\left[\frac{u}{2}\mathrm{erfc}\left(\frac{u}{V_1}\right) - \frac{V_1}{2\sqrt{\pi}}\exp\left(-\frac{u^2}{V_1^2}\right)\right], \tag{8}$$

$$\frac{1}{4} = C_2 - n_1\beta\left[\left(\frac{u^2}{2} + \frac{V_1^2}{4}\right)\mathrm{erfc}\left(\frac{u}{V_1}\right) - \frac{V_1 u}{2\sqrt{\pi}}\exp\left(-\frac{u^2}{V_1^2}\right)\right], \tag{9}$$

$$\frac{1}{\sqrt{\pi}} = \beta n_1 V_1^3\left[\frac{1}{2\sqrt{\pi}}\left(\frac{u^2}{V_1^2} + 2\right)\exp\left(-\frac{u^2}{V_1^2}\right) - \frac{u}{V_1}\frac{1}{2}\left(\frac{5}{2} + \frac{u^2}{V_1^2}\right)\mathrm{erfc}\left(\frac{u}{V_1}\right)\right] + C_3. \tag{10}$$

Equations (5), (5′), (6), (6′), (8), and (9) are identical to the corresponding mass and momentum conservation equations obtained in Refs. 11–18, whereas Eqs. (7), (7′), and (10) differ from the corresponding energy conservation equations[11–18] by the temperature gradient term, which is responsible for conduction heat flux to the ablative surface.

Let us introduce a thermal conduction parameter τ_T

$$\tau_T = \frac{V_T V_1}{\nu}\frac{d(\ln T)}{dx} = \frac{\lambda_{\mathrm{mfp}}}{\delta x_T} \ll 1, \tag{11}$$

where $\lambda_{\mathrm{mfp}} = (V_T \cdot V_1)/\nu$ is the gas mean-free-path at the outer boundary of the kinetic layer and $\delta x_T = [d(\ln T)/dx]^{-1}$ is the characteristic gradient length. Condition (11) is needed for the Chapman–Enskog expansion method and Eq. (7) to be valid, as explained earlier. In the case of small u, Eqs. (8)–(10) can be simplified to the following form:

$$V_1 = 1 - u\frac{\sqrt{\pi}}{8} + \frac{5\sqrt{\pi}\tau_T}{8}, \tag{12}$$

$$\beta = 1 + u\left(\frac{2}{\sqrt{\pi}} - \frac{9\sqrt{\pi}}{16}\right) + \frac{5\sqrt{\pi}\tau_T}{16}, \tag{13}$$

$$n_1 = 1 - u\left(\frac{2}{\sqrt{\pi}} + \frac{5\sqrt{\pi}}{16}\right) - \frac{15\sqrt{\pi}\tau_T}{16}. \tag{14}$$

As we can see, for $u=0$ (in which case there is no ablation) and for $\tau_T > 0$, the temperature and density at the outer edge of kinetic layer are correspondingly larger and smaller than 1; it is as expected, as the gas bulk region has a higher temperature than the wall surface and the total conduction heat flux is assumed to be directed to the wall ($\tau_T > 0$). These

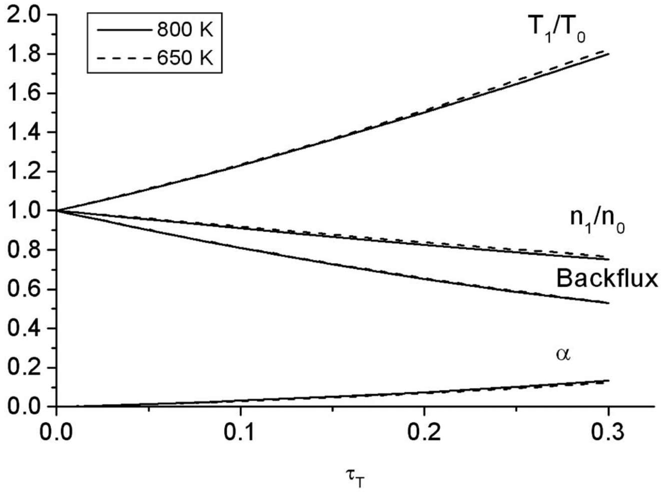

FIG. 2. Normalized u, $T_1 = V_1^2$, and back flux at the ablative surface vs τ_T.

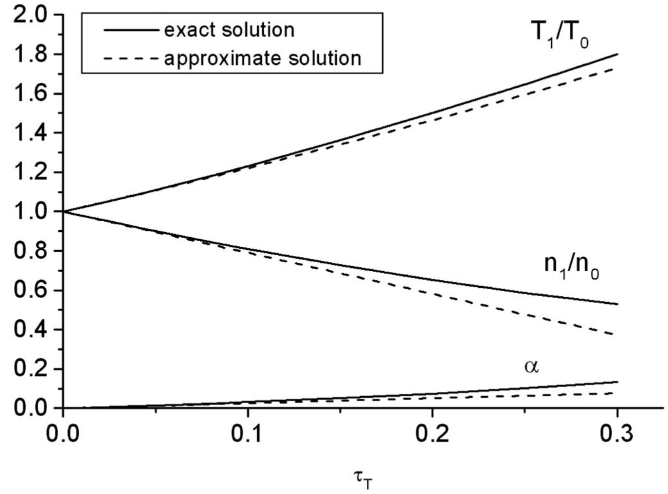

FIG. 3. Comparison of exact and approximate solutions. Polyethylene with a surface temperature of 800 K.

temperature jumps between the solid surface and the Maxwellian gas at the outer edge of the kinetic layer is a very well known phenomenon in gas dynamics, see, e.g., Ref. 24.

Assuming that the ablation process is due to only the gas thermal conduction (no external heat source is applied to the ablative surface), the boundary condition at the ablative surface can be written as

$$-E_x - \kappa_{\mathrm{wall}} \cdot \frac{dT_{\mathrm{wall}}}{dx} = M_x \cdot \Phi_{\mathrm{vap}}, \tag{15}$$

where κ_{wall} is thermal conductivity of the wall, Φ_{vap} is evaporation heat of the wall material, $-E_x$ is the total energy flux through the Knudsen layer incoming into the ablative wall, Eq. (7′), and M_x is the mass ablation rate, Eq. (5′); the negative sign in front of E_x is due to the x axis being directed from the wall into the gas chamber. Assuming no heat loss in the bulk of the wall, the boundary condition at the ablative surface is reduced to:

$$-E_x = M_x \cdot \Phi_{\mathrm{vap}}. \tag{16}$$

This can be the case of a "boiling wall," where all heat incoming into the wall is spent on vaporization. Thus, for a given ablative surface temperature and corresponding equilibrium vapor pressure and number density, and given heat conduction parameter τ_T, Eqs. (8)–(10) and (15) can be solved relative to variables n_1, V_1, u, and β. The total ablation rate and the heat flux to the ablative wall [Eqs. (5′) and (7′)] can then be computed. The dependences of n_1, V_1, and back flux (the total flux of particles incoming to the ablative surface from the gas) on the τ_T and for a specific example of polyethylene wall are presented in the following.

III. RESULTS

Figure 2 shows the calculated parameters of the Knudsen layer versus τ_T for the case of thermal conduction heating of the ablative polyethylene wall with the polyethylene surface temperatures of 650 and 800 K, Eqs. (8)–(10) and (16); the evaporation heat has been taken as 3.6×10^6 (J/kg) and equilibrium vapor pressure equal as $P = 10^5 \exp[5565.22(1/453 - 1/T)]$,[25] where the pressure is in pascal and temperature is in kelvin. One can see that u, T, back flux, and n_1 are weak functions of temperatures in this temperature region, although their equilibrium vapor pressures differ almost by five times. As it has been mentioned earlier, we cannot extend the obtained results of Fig. 2 for higher τ_T, because the Chapman–Enskog expansion method is valid only for $\tau_T \ll 1$. In this limit, Eq. (16) can be simplified to the following form:

$$-u\left(\frac{\Phi_{\mathrm{vap}} m}{kT_0}\right) = \frac{5}{2}u - \frac{5}{4}\tau_T, \tag{17}$$

where m is the average atomic mass of polyethylene composition and T_0 is the polyethylene surface temperature. Expressing u as a function of τ_T, Eq. (17), and substituting it into Eqs. (12) and (14) and then into the following equation for normalized back flux at the ablative wall:

$$F_{b\text{-flux}} = 1 - un_1 2\sqrt{\pi} \approx 1 - u2\sqrt{\pi}, \tag{18}$$

yields the explicit relationships between n_1, $T = V_1^2$, u, $F_{b\text{-flux}}$, and τ_T. Comparison of this approximate solution and the "exact" solution obtained from the solution of Eqs. (8)–(10) is shown in Fig. 3, leading to the conclusion that approximate solution gives satisfactory results for $\tau_T < 0.05$.

We also would like to point out that in the case of thermal conduction, u is small and cannot reach sonic condition. Otherwise, E_x would be positive, Eqs. (7) and (7′), meaning that the total energy flux would be directed not to the ablative surface but upward into the gas chamber, contradicting the model assumptions.

In the case where the ablating surface is heated by an additional (external) heat source, e.g., by laser radiation,[9] Eq. (16) can be rewritten as

$$-E_x - E_{\mathrm{ext}} = M_x \Phi_{\mathrm{vap}}, \tag{19}$$

where E_{ext} is an external heat flux to the ablative surface. It is worth noting that in the case where thermal conduction in the wall is not equal to zero, see Eq. (15), the E_{ext} is the net external heat flux at the wall equal to the external heat flux to the wall "above" the surface minus conduction heat into the

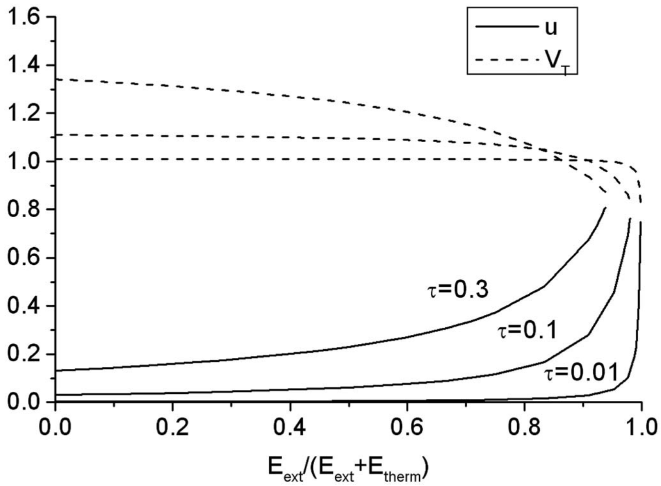

FIG. 4. Normalized thermal and directed velocities at the outer boundary of the Knudsen layer as functions of ratio of external heat flux to the total heat flux at the ablating polyethylene surface.

wall "behind" the surface. With an increase in E_{ext} the ablation rate increases and, if E_{ext} becomes much larger than conduction heat flux, we may drop the conduction heat term from the velocity distribution function [Eq. (3)], recovering the previous models.[9–15] Figure 4 shows V_1 and u as functions of the ratio of an external heat flux to the total heat flux, $q=E_{\text{ext}}/(E_{\text{ext}}+E_{\text{thermal}})$ for the ablative polyethylene wall with the polyethylene surface temperatures of 800 K and for $\tau_T=0.01$, 0.1, and 0.3; the calculations have been performed up to sonic conditions with $\gamma=5/3$. As one can see with a decrease in τ_T and an increase in q, the distributions of V_1 and u are converging to the case of $\tau=0$.

The effect of heat conduction in the case of an external heat flux can also be calculated, following,[10–15] as functions of $\alpha=u/V_1$ for different τ_T, Eqs. (8)–(10); here, the variation of α indicates the magnitude of the external heat flux compared to the conduction heat flux (as α and the flow velocity increase up to sonic condition, there can be no heat conduction to the wall). The results of these calculations are shown in Fig. 5. As one can see at small α the effect of thermal conduction is important, and with an increase in α, the effect of thermal conduction decrease, E_x increases and changes sign [becomes positive meaning that the total energy flux is directed upward, Fig. 1, into the gas chamber, see Eqs. (7) and (7′)] and the temperature and density plots are converging to the case of $\tau_T=0$. As one can see, T_1 and n_1 decreases with an increase in α (with an increase in directed velocity u), Fig. 5, that can be explained by expansion of the dense ablated gas stream in less dense gas surrounding. It is worth noting, that such a decrease in T_1 and n_1 leads to a decrease in the thermal heat conduction at a given τ_T, see Eqs. (7) and (11).

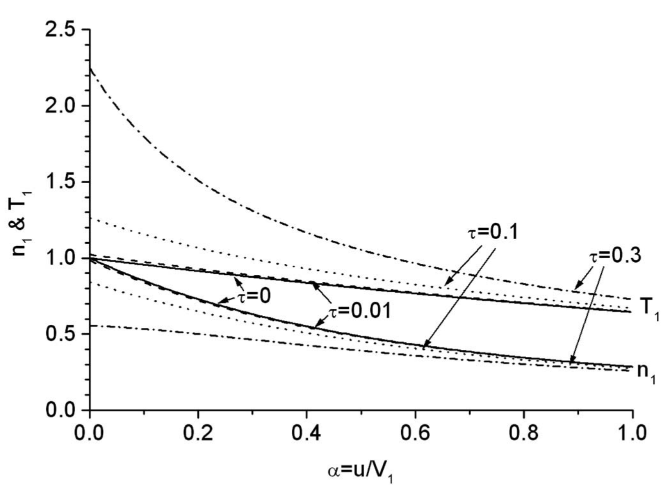

FIG. 5. Normalized temperature and density at the outer boundary of Knudsen layer as a function of $\alpha=u/V_1$ with temperature gradient at the Knudsen layer edge as a parameter.

IV. CONCLUDING REMARKS

In summary, a model of the Knudsen layer near the ablated surface in the case of surface heating by the adjacent gas, by virtue of thermal conductivity, was developed. Previously existing models were not able to describe this physical situation due to neglecting effect of thermal conductivity on the velocity distribution function in the gas and thus leading to temperature gradient directed outwards the surface. In contrast, the developed model predicted existence of the temperature gradient toward the surface. It should be pointed out that this model is limited to relatively small temperature gradients due to the limitation of the Chapman-Enskog expansion for solving the Boltzmann equation. In the case of a larger temperature gradient a more rigorous model is required and only numerical simulations such as DSMC would be able to solve the problem. Finally, the model can be also verified experimentally, e.g., in liquid–vapor experiments, where the surface temperature, the mass flux, and the bulk gas pressure are measured as in Ref. 26. Unfortunately, we cannot use this work, because their experiments have been performed on "bending" liquid–vapor interface. As shown in this article and in Ref. 20, the measured high temperature jump in this experiment is, probably, due to very small curvature radius of the vapor–liquid interface. To the best of our knowledge there currently is no experimental data that can allow direct experimental verification of the model, and comparison with direct numerical simulations will play a similar role until a more complete physical model can be developed.

[1]M. Seeger, L. Niemeyer, T. Christen, M. Schwinne, and R. Dommerque, J. Phys. D **39**, 2180 (2006).
[2]M. Keidar and I. I. Beilis, Phys. Plasmas **13**, 114503 (2006).
[3]R. Burton and P. Turchi, J. Propul. Power **14**, 716 (1998).
[4]M. Keidar, I. D. Boyd, and I. I. Beilis, IEEE Trans. Plasma Sci. **28**, 376 (2000).
[5]M. I. Boulos, P. Fauchais, and E. Pfender, *Thermal Plasmas: Fundamentals and Applications* (Plenum, New York, 1995), Vol. 1.
[6]I. I. Beilis, IEEE Trans. Plasma Sci. **34**, 885 (2006).
[7]L. L. Raja, P. L. Varghese, and D. E. Wilson, J. Thermophys. Heat Transfer **11**, 353 (1997).
[8]L. V. Zhigilei, P. B. S. Kodali, and B. J. Garrison, J. Phys. Chem. B **102**, 2845 (1998).
[9]S. I. Anisimov, Sov. Phys. JETP **27**, 182 (1968).
[10]T. Ytrehus, Rarefied Gas Dyn. **51**, 1197 (1976).
[11]I. I. Beilis, IEEE Trans. Plasma Sci. **13**, 288 (1985).
[12]I. I. Beilis, *Vacuum Arc Science and Technology*, edited by R. L. Boxman, P. Martin and D. Sanders (Noyes, Park Ridge, NJ), 2008 (1995).

[13]M. Keidar, J. Fan, I. D. Boyd, and I. I. Beilis, J. Appl. Phys. **89**, 3095 (2001).
[14]M. Keidar, I. D. Boyd, and I. I. Beilis, J. Phys. D **34**, 1675 (2001).
[15]M. Keidar, I. D. Boyd, and I. I. Beilis, J. Appl. Phys. **96**, 5420 (2004).
[16]D. Sibold and H. M. Urbassek, Phys. Rev. A **43**, 6722 (1991).
[17]D. Sibold and H. M. Urbassek, Phys. Fluids A **5**, 243 (1993).
[18]A. A. Morozov, Appl. Phys. A: Mater. Sci. Process. **A79**, 997 (2004).
[19]I. N. Shishkova and S. S. Sazhin, J. Comput. Phys. **218**, 635 (2006).
[20]J. W. Rose, Int. J. Heat Mass Transfer **43**, 3869 (2000).
[21]M. Bond and H. Struchtrup, Phys. Rev. E **70**, 061605 (2004).
[22]W. G. Vincenti and C. H. Kruger, Jr., *Introduction to Physical Gas Dynamics* (Krieger, Malabar, FL, 1975).
[23]O. Knacke and I. N. Stranski, Prog. Met. Phys. **6**, 181 (1956).
[24]E. H. Kennard, *Kinetic Theory of Gases* (McGraw–Hill, New York, 1938).
[25]M. Keidar and I. D. Boyd, J. Appl. Phys. **99**, 053301 (2006).
[26]C. A. Ward and D. Stanga, Phys. Rev. E **64**, 051509 (2001).